## Preamble and Acknowledgments

One of the major tasks of geodesy is the determination of the geoid, which is defined an equipotential surface the earth gravity field which coincides on average with mean sea level. According to *C.F. Gauss*, the geoid is the "mathematical figure of the Earth", and in fact, of the gravity field. The geoid surface is more irregular than the ellipsoid of revolution often used to approximate the shape of the physical Earth, but considerably smoother than the Earth's physical surface. While the latter has excursions of the over of 8 km (Mount Everest) and −11 km (Mariana Trench), the geoid varies only about ±100 m about the reference ellipsoid of revolution.

The geoid is basic information in different disciplines related to *Geomatics* science from navigation, mapping and surveying to construction, detecting the variations of the ocean currents, study of the interior properties of the Earth in geophysics, seismology and plate tectonics. Nowadays geoid determination is getting even more crucial, due to the development of the *Global Navigation Satellite Systems* (GNSS). These systems can offer three-dimensional positioning over the world, but without having the precise geoid model, in the field of engineering the system can be used only for two-dimensional positioning. This is due to the fact, that global positioning systems provide ellipsoidal heights, which are geometric heights, instead of orthometric heights, which have physical meanings. In order to convert the ellipsoidal height into a more useful orthometric height we need to know the geoid undulation at the station.

New developments and advances in the gravity field determination from satellite tracking have taken place in the last few years. Recent satellite gravity missions, such as the CHAllenging Minisatellite Payload (*CHAMP*), Gravity Recovery and Climate Experiment (*GRACE*) and Gravity field and steady-state Ocean Circulation Explorer (GOCE) have provided global scale and very accurate gravity data. However, the up-to-date, accuracy of the global geoid modelling is a few decimetres, which is not sufficient for many scientific and engineering applications. So, high resolution regional geoid models are still necessary for the most practical purposes.

Many different methods have been proposed through the years for regional geoid determination by gravimetric data, each using its own set of techniques and philosophy. Today, all such methods combine long-wavelength Earth Gravity Models (EGMs) with local gravity data, and they mainly differ in the way they combine these data sets. The method, called Least-Squares Modification of Stokes (LSMS) Formula with Additive corrections (LSMSA), is the result of 30 years of research and numerous Ph.D. theses at KTH. The LSMSA is an accurate, simple and practical method of determining the geoid. The theoretical and practical aspects of this method have been developed since 1984 to present mainly by or under the supervision of Prof. Lars E. Sjöberg. (See numerous researches, e.g. in J. of Geodesy.) The method has been successfully applied in the determination of several high-resolution regional geoid models in different areas. Through the LSMSA approach (KTH method), different heterogeneous data, e.g., a Global Geopotential Model, gravity anomalies, a high-resolution photogrammetric/SRTM Digital Elevation Model, and the method can be (and usually is) designed to match with GPS/levelling data by using the least-squares principle. This method is unique in that it uses least squares technique in the spectral domain to combine the data in an optimum way by considering the errors of the EGM, the gravity data and the truncation of Stokes' integral to a cap around the computation point.

Another feature that distinguishes the KTH method form others is the way corrections for topography, atmosphere and ellipsoidal shape of the earth are applied: in contrast to other methods that apply these corrections both to the gravity anomaly and to the preliminary computed geoid heights, we only correct the preliminary geoid heights by so-called additive corrections. Any of the additive corrections can add afterward at any time when better data calls upon its improvement (without the need to repeat all the computations). In the recent test project for the comparison of up-to-date methods of geoid modelling with data from Auvergne area in France, no method provided better results than the LSMSA (http://w3.uniroma1.it/Hotine-marussi_symposium_2009/SubAbs.asp).

The main purpose of this book is to test the potential of the Royal Institute of Technology (KTH) approach based on the Least-Squares Modification of Stokes (LSMS) formula (Sjöberg, 1984, 1991, 2003c and 2003d) as an optimum way for combination and determination of a new high-resolution geoid model for Iran.

Through the book, different theoretical and practical aspects in gravity modelling were studied. The most highlighted subjects are:

- Digital Elevation Models (DEM) and their applications in gravity modelling
- Global Geopotential Models and their rules in gravity modelling

- Geoid and Quasi-geoid modelling
- Crustal depth modelling
- Satellite  altimetry and sea surface topography modelling

*Modern techniques in gravity modelling* have been written as a comprehensive textbook that guides the reader through the theory and practice of gravity modelling. It can be used as a practical manual for determination of gravimetric geoid. The idea for this book emerged when we realized that documentation on the methods, models and tools of gravity modelling was either spread over numerous technical and scientific publications accessible to a wider community.

The book is composed of five chapters which reflected the current state-of-the-art achievements and the future research direction in the field. The collection of topics aims to reflect the diversity of recent advances in gravity modelling. We hope this book will be relevant and valuable to the whole geosciences community and serve to both define and advance the state of geodesy. Each part addresses supplementary topics of practical interest.

*Part One* presents the mathematical aspects of the KTH approach to determine the geoid based on the LSMS formula, including its additive correction terms. The Stokes method and the mathematical model for the determination of the precise regional gravimetric geoid are fully described in this chapter. In *Part two*, we explain some practical aspects related to terrestrial gravity data, Global Geopotential Models and Digital Elevation Models. A complete numerical case study for quasi-geoid modelling, will be illustrated in *Part three.* The internal accuracy of the geoid model was estimated by means of the expected global mean square error, whereas the GPS-levelling data is applied for an external evaluation of the accuracy of the computed geoid models.

*Part four* we includes inverse gravity modelling approach for estimation of crustal depth model. Finally, in Part *six* we explain the procedure for handing of satellite altimetry data in modelling of sea surface topography and mean sea level.

*Part four* presents the procedure for quasi-geoid modelling based on the KTH approach.

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*Ramin Kiamehr*

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